https://www.proconference.org/index.php/gec/article/view/gec41-00-038
DOI: 10.30890/2709-1783.2025-41-00-038

УДК 539.3

ASYMPTOTIC ASSESSMENT OF LOADS IN THE WIND TURBINE BLADE ATTACHMENT TO THE HUB

АСИМПТОТИЧНА ОЦІНКА НАВАНТАЖЕНЬ У ВУЗЛІ З’ЄДНАННЯ ЛОПАТЕЙ ВІТРЯКА ТА ХАБУ

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Abstract. *The efficiency and durability of wind energy systems largely depend on design solutions and the reliability of their connections. One of the key elements is the contact interaction between the hub and the blade. This study examines the application of the perturbation method [1, 2] to investigate the mechanical interaction between a composite material hub and a wind turbine blade. The results may be useful for improving reliability and optimizing the design of wind energy systems.*

Keywords. *Asymptotic analysis, perturbation method, mathematical model, anisotropic plate, curvilinear anisotropy, wind turbine, blades, hub, bolt.*

Анотація. *Розглянуто модельну задачу контактної взаємодії жорсткого хабу та лопаті вітротурбіни, яка виготовлена з композиційного матеріалу з циліндричною анізотропією. Це означає, що механічні властивості матеріалу змінюються залежно від кута та радіального положення, що характерно для шаруватих або намотаних композитів. Болт вважається абсолютно жорстким. До нього прикладено зовнішнє зусилля , що може включати осьову силу, згинальний момент або комбінацію навантажень. Хаб виготовлений з композитного матеріалу, механічні властивості якого змінюються в залежності від радіальної координати та кута (циліндрична анізотропія). Контакт між хабом і болтом жорсткий. Контактний тиск буде неоднорідним через анізотропію матеріалу. Задача розв’язана асимптотичним методом Маневича-Павленко. Отримано асимптотичний розв’язок у нульовому наближенні. Раніше в роботах було показано, що саме цей розв’язок для ізотропних матеріалів співпадає з відомим точним. Отримані результати можуть бути використані для оптимізації конструкції з'єднання хабу і лопаті та прогнозування довговічності матеріалу. Одержані на основі запропонованого підходу значення для контактних напружень разом з особливим розв’язком дають рівномірно придатний в усій області контакту наближений розв’язок задачі для ефективного дослідження картини контактної взаємодії зазначених елементів конструкції сучасного вітряка.*

Ключові слова. *Хаб, вітряк, асимптотичний метод, анізотропія, ортотропний, пластина*

Introduction.

The solution of the contact problem for an orthotropic plate with cylindrical anisotropy and a stringer under their rigid contact, obtained by the author using the analytical asymptotic Manevich-Pavlenko method, can be applied in various practical problems of civil engineering, shipbuilding, and others. In this article, we will examine for the first time the application of the results of this solution for studying the contact of certain elements of a wind turbine structure. The contact interaction of the blades and the hub made of composite material, which are connected with bolts, is considered. The hub of the wind turbine is the central part to which the blades are attached. Its strength and weight critically affect the efficiency and durability of the wind installation. In modern wind turbines, the attachment of the blades to the hub is carried out using high-strength bolts, as this ensures reliability and the possibility of maintenance. The blade-to-hub connection points are a very important component of the structure. To avoid energy loss at such joints, to reduce the weight of the structure, to maintain its strength and service life, it is necessary to study the mechanical properties of their interaction (tension, shear, stress in the contact area) and to look for new materials for the structural elements that will allow for savings.

Material and research results. The main types of blade attachment to the hub using bolts are, firstly, the ring flange mounting – the blade has a flange that is fastened to the hub with bolt connections. It is used in large turbines (2+ MW). The main advantage is the uniform distribution of load. Insert-type bolt connections are still actively used. Bolts are screwed into insert metal sleeves integrated into the root part of the blade, resulting in less load on individual attachment points. Traditionally, hubs are made from steel or cast iron with high strength characteristics. However, due to the heavy weight of these materials, the load on the supporting elements of the wind turbine increases, which affects the overall system efficiency. The advantage of composite materials is their low weight and corrosion resistance, which is especially important for offshore wind turbines.

Main text.

Hubs made from composite materials (polymer composites reinforced with carbon or glass fiber) are still rare today, mostly only found in experimental or small turbines, but they have certain advantages that need to be investigated. Let us consider a model problem of the contact interaction between a rigid hub and a wind turbine blade made from a composite material with cylindrical anisotropy. This means that the mechanical properties of the material vary depending on the angle and radial position, which is characteristic of laminated or wound composites. The bolt is considered to be perfectly rigid. An external force is applied to it, which may include axial force, bending moment, or a combination of loads. The hub is made of a composite material, whose mechanical properties vary depending on the radial coordinate and angle (cylindrical anisotropy). The contact between the hub and the bolt is rigid. The contact pressure will be non-uniform due to the material anisotropy. That is, the problem setup will be identical to the following problem of transferring load from a rod to an orthotropic plate with cylindrical anisotropy, shaped like a ring sector (Fig. 3). Spring plate $R_0 \le r < \infty, \quad -\gamma \le \theta \le \gamma$ secured along the edges $\theta = \pm\gamma$. At infinity, stress and displacement are absent. Along the median radius ($\theta = 0$) the plate is reinforced with a rod, which at the extreme point $r = R_0$ subjected to axial load P_0 (in this task, the direction was considered opposite to the direction of the bolt pressure on the hub).

Figure 1 - Node design.

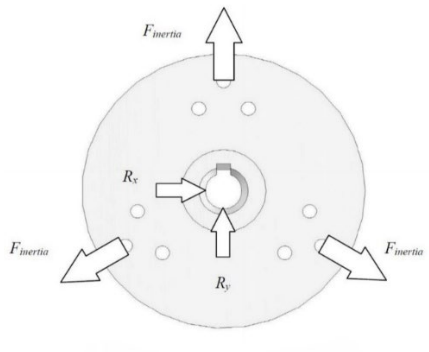

Figure 2 - Load on the hub.

It is considered that $0 \le \xi \le h;\ h = \ln R_1 / R_0;\ (R_0 \le r \le R_1);\ 0 \le \eta \le \gamma.$

This problem reduces to integrating the equilibrium equations of the plate in terms of displacements

$$\begin{cases} B_1 u_{\xi\xi} + G u_{\eta\eta} - B_2 (v_\eta + u) + G m v_{\xi\eta} - G v_\eta = 0, \\ G v_{\xi\xi} + B_2 v_{\eta\eta} + B_2 u_\eta + G m u_{\xi\eta} + G (u_\eta - v) = 0; \end{cases} \quad (1)$$

under such boundary conditions:

$$\sigma_1 = B_1\left(R_0 e^{\xi}\right)^{-1}\left(u_{\xi} + \vartheta_2(v_{\eta} + u)\right) = 0,\ \tau = G\left(R_0 e^{\xi}\right)^{-1}\left(u_{\eta} + v_{\xi} - v\right) = 0\ (\xi = 0, \xi = h),$$

$$u = u_c, v = 0 \quad (\eta = 0); u = v = 0 \quad (\eta = \pm\gamma). \tag{2}$$

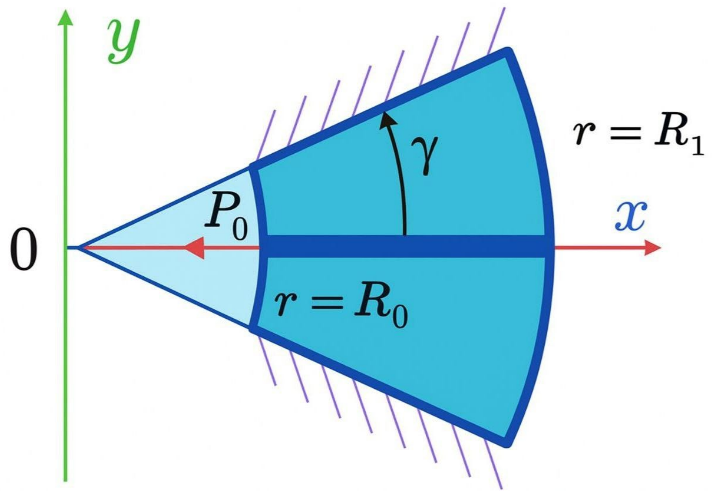

Figure 3- Contact scheme.

Rod displacement u_c satisfies the ratio

$$EFu_{c\xi\xi} = P_0 R_0 e^{\xi}\delta(\xi) - 2R_0 e^{\xi}\tau(\xi, 0) \tag{3}$$

and the contact interaction force between the rod and the plate is determined by the formula (4).

$$T(\xi) = \tau(\xi, 0) = G\left(R_0 e^{\xi}\right)^{-1} u_{\eta}\big|_{\eta=0}. \tag{4}$$

In the first approximation for this problem, the first type of boundary value problem has the form (first type stress state)

$$B_1 u_{\xi\xi}^{1,0} + G u_{\eta\eta}^{1,0} = 0, \tag{5}$$

$$v_{\eta}^{1,0} + u^{1,0} = 0. \tag{6}$$

with boundary conditions $u^{1,0} = u_c\ (\eta = 0);\ \ u^{1,0} = 0\ (\eta = \gamma)$. (7)

Equations corresponding to the second type of NDS have the form

$$G v_{\xi\xi}^{2,0} + B_2 v_{\eta\eta}^{2,0} = 0, \quad u_{\xi\xi}^{2,0} = 0, \tag{8}$$

under such boundary conditions:

$$v_{\xi}^{2,0} = -u_{\eta}^{1,0} \ (\xi = 0; \xi = h), \ v^{2,0} = -v^{1,0} \ (\eta = 0; \eta = \gamma) \tag{9}$$

For the first-type NDS, the components of the displacement vector are obtained

$$u^{1,0}(\xi,\eta) = -\frac{P_0 R_0}{EF}\frac{\omega(\gamma-\eta)}{gh} - \frac{2P_0 R_0}{EFh}\sum_{n=1}^{\infty}\frac{f(n,\eta)\cos(\alpha_n \xi)}{\varphi(n)}, \tag{10}$$

$$v^{1,0}(\xi,\eta) = -\frac{P_0 R_0}{EF}\frac{\omega}{gh}\frac{(\gamma-\eta)^2}{2} - \frac{2P_0 R_0}{\omega EFh}\sum_{n=1}^{\infty}\frac{f_1(n,\eta)\cos(\alpha_n \xi)}{\alpha_n \varphi(n)} \tag{11}$$

where $\alpha_n = n\pi/h$; $f(n,\eta) = sh[\omega\alpha_n(\gamma-\eta)]/sh(\omega\alpha_n\gamma)$; $g = 2G\omega/(EF)$,

$f_1(n,\eta) = ch[\omega\alpha_n(\gamma-\eta)]/sh(\omega\alpha_n\gamma)$; $\omega = (B_1/G)^{1/2}$; $\varphi(n) = \alpha_n^2 + g\alpha_n cth(\omega\gamma\alpha_n)$.

$$v^{1,0}(\xi,0) = -\frac{P_0 R_0}{EF}\frac{\omega\gamma^2}{2gh} - \frac{2P_0 R_0}{EF}\frac{h^2}{\omega\pi^3}\sum_{n=1}^{\infty}\frac{cth(\omega\alpha_n\gamma)\cos(\alpha_n\xi)}{n^2[n + g(h/\pi)cth(\omega\alpha_n\gamma)]},$$

$$v^{1,0}(\xi,\gamma) = -\frac{2P_0 R_0}{EF}\frac{h^2}{\omega\pi^3}\sum_{n=1}^{\infty}\frac{\cos(\alpha_n\xi)}{n^2 sh(\omega\alpha_n\gamma)[n + g(h/\pi)cth(\omega\alpha_n\gamma)]}$$

Conclusions.

The obtained solutions do not satisfy the zero boundary conditions for the tangential stress τ for the plate at $\xi = 0$ i $\xi = h$, as well as zero boundary conditions for the component of the displacement vector v at $\eta = 0$ and $\eta = \gamma$. Elimination of the corresponding inconsistencies is ensured by solving the second-type boundary value problem of the non-stationary heat conduction. It should be noted that the solutions for the contact interaction force obtained by the asymptotic method are valid everywhere except in the immediate vicinity of the points. $\xi = 0, \eta = 0$ and $\xi = 0, \eta = \gamma$, where it is necessary to use a particular solution $\tau(x) = Ax^{-\lambda}$.

An unknown constant coefficient is determined from the ‘matching’ conditions (at a certain point, the particular and approximate solutions, as well as their derivatives, coincide). These conditions allow for determining the matching point of the two solutions and the constant of the particular solution. It has been shown that the zone in which it is necessary to use the particular solution is insignificant. The obtained results can be used for optimizing the design of the hub-blade connection and predicting the material's durability. The values obtained based on the proposed approach for contact stresses, together with the particular solution, provide an approximate solution to the problem that is uniformly applicable throughout the contact area, allowing for an effective study of the contact interaction pattern of the specified elements of a modern wind turbine construction.

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Статтю надіслано: 24.10.2025 р.