MATHEMATICAL MODEL FOR THERMAL CALCULATION OF VEHICLE DISC BRAKES ON 1ST TYPE TESTS

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The article presents the bases of thermal processes mathematical modeling in vehicle disc brakes that in combination with the experiment planning method allowed a complex approach to their thermal calculation.

Key words: motor vehicle (MV), 1st type test, mathematical modeling, theories of experiment planning, thermal models

Introduction

Braking is a form of motor vehicle (MV) driving that ensures speed reduction and its motionless positioning. The vehicle braking properties are specified by the Regulations No. 13 of the Committee on Internal Transport of the United Nations Economic Commission for Europe [1]. Special interest is generated by the study of the brakes thermal behavior at cycling braking specified by the 1st type tests in accordance with the given regulations.

Until recently, the studies of the brake units thermal behavior were conducted with the help of the methods of single-factor or sequential experiments. The results of such studies are presented in the form of a great variety of diagrams, on the basis of which there are developed criteria dependencies, in accordance with which it is impossible to determine numeric temperature values when reuniting the factors that determine them [2, 3, 4, 5]. Therefore, the issue of development of efficient study methods on the basis of the similarity, modeling, and experiment planning theories becomes topical since these methods provide a possibility to solve a lot of the fundamentally new problems that cannot be resolved with the help of the common classical ones (methods of one of the theories).

Study Materials

In the theory of thermal conductivity, this problem is formulated as both-side heating of the unlimited plate with the thickness of 25 with the help of the constant specific heat flow $Q$. The thermal conductivity equation for our case is presented together with the boundary conditions in the work [6]. The solution variants of this boundary problem that were obtained by some method or other are also known [7, 8].

However, when modeling the preliminary stage of the 1st type tests, there arises a necessity to disconnect disc brake friction pairs after each of the MV 20 braking cycles of the M3 category.

When determining the temperature fields in disc brakes with the help of calculation, it is required to solve the equation in partial derivatives that describe the...
processes of heat transfer in complex objects with the distributed parameters under the corresponding boundary conditions [6]:

\[
\lambda(x, y, z) \frac{\partial^2 T}{\partial x^2} + \lambda(x, y, z) \frac{\partial^2 T}{\partial y^2} + \lambda(x, y, z) \frac{\partial^2 T}{\partial z^2} + Q(x, y, z) = c\rho(x, y, z) \frac{\partial T}{\partial \tau},
\]

where \(T\) – temperature;
\(x, y, z\) – current coordinates of the brake unit;
\(\lambda(x, y, z)\) – thermal conductivity coefficient;
\(c\rho(x, y, z)\) – volumetric thermal capacity;
\(Q(x, y, z)\) – thermal flow density;
\(\tau\) – process time.

Cooling of the brake unit friction pairs when they are disconnected is described by the equation:

\[
\lambda(x, y, z) \frac{\partial^2 T}{\partial x^2} + \lambda(x, y, z) \frac{\partial^2 T}{\partial y^2} + \lambda(x, y, z) \frac{\partial^2 T}{\partial z^2} = c\rho(x, y, z) \frac{\partial T}{\partial \tau}.
\]

This problem doesn’t have an accurate analytical solution since it is one of the non-stationary contact thermal problems in the zones of non-classical form under inhomogeneous boundary and complex initial conditions that are peculiar to cycling braking.

**Calculation Module**

To solve the equations (1) and (2), there was used the calculation module [5] developed on the basis of the software system “Fourier – 2 x, y, 2” that allowed to solve two-dimensional and three-dimensional heat transfer problems interactively and obtain results in the user-friendly and visual form.

The program allows entering initial conditions separately into each array unit. Besides, in case of homogeneous temperature distribution, the program provides for their automatic entry in the whole array from the upper corner of the temperature array.

After the set period of braking time, the initial temperature distribution \(T_0\) changes and new temperature distribution begins in the disc at the beginning of the cooling process that is stored as the solution result. Temperature redistribution in the elements of friction pairs takes place during the cooling process. Before the repeated braking begins, there will be new initial conditions that will be also stored as the solution results.

The initial or time conditions determine the initial thermal state of the studied bodies. Therefore, the aim of the initial conditions is to assign the temperature distribution inside the body at the initial time:

\[
T(x, y, z, 0) = T_0 = T(x, y, z).
\]

The temperatures at the beginning of braking and brake releasing (at the beginning of heating and cooling) are variable, i.e. there are difficult initial conditions, at the intermittent service and comparatively long braking on the descent and ascent routes that alternate.

In case when the special operating modes are considered, there can be accepted the homogeneous temperature distribution at the initial time. Then
\[ T(x, y, z, 0) = T_o = T_B = \text{const}, \]  
where \( T_B \) – environmental temperature.

As a rule, the boundary conditions of the first, second, and third kind are studied. The boundary conditions of the first kind consist in assignment of the body surface temperature at any time:

\[ T_n(\tau) = f(\tau), \]  
where \( T_n \) – surface temperature.

The boundary conditions of the second type characterize the law of heat emission on the friction surfaces. Therefore, the aim of the boundary conditions of the second type for the brake unit consists in assignment of the thermal flow density for each friction surface point as a time function:

\[ q_{II}(\tau) = f(\tau), \]  
where \( q_{TF} \) – thermal flow density.

The boundary conditions of the third type characterize the law of heat convection between the body surface and environment. Since this law is very complex, we accept that, in order to simplify the problem, it is described by the Newton’s formula:

\[ q_{II}(\tau) = \alpha \left[ T_n(\tau) - T_B(\tau) \right]. \]  

Based on the equality of the added and removed heat:

\[ \alpha \left( \frac{\partial T}{\partial n} \right) = \alpha \left[ T_n(\tau) - T_B(\tau) \right] = 0. \]

Thus, in order to realize the boundary conditions of the third kind, it is necessary to have the data about the coefficient of heat transfer \( \alpha \) and environmental temperature.

**Study of the Main Models**

The program provides for assignment of environmental temperature and coefficient of heat transfer from the upper corner throughout the whole array and rows from the first column of the corresponding arrays. As it was mentioned above, the assignment of the boundary conditions of the third kind is realized in all the model corners during the cooling process.

Assignment of the time periods is determined by the number of wheel rotations per second. The time increment \( \Delta \tau \) at the brake lining width that is equal to the disc quadrant should comply with the time that is necessary for the wheel to make a quarter turn. The aim of the period \( \tau \) is determined by the number of wheel rotations that should be made before temperature determination. Thus, when modeling, the temperature value can be determined through a wheel quarter turn or any time period \( \tau \) during the processes of braking and cooling.

The heat emission source shifts one quadrant against the rotation direction in accordance with the inversion method principle \([5]\) during the process of making a solution after the time that determines a wheel quarter turn that depends on the motion speed.

The new approach to modeling of the previous stage of the 1\textsuperscript{st} type tests consists in its implementation in three models: model of the brake assembly heating, model of the brake lining assemblies cooling, and model of the brake disc cooling (see fig. 1)
Fig. 1. Configuration files of the brake unit heating (a), brake lining cooling (b), and brake disk cooling (c)

The sector that is developed on the two-dimensional grid with the sufficient level of accuracy is modeled on the grid in accordance with the coordinate z. Hereewith, the changes of the average sector thickness in accordance with the coordinate y with the angle Δy is taken into account by the change of the thermophysical coefficients for each horizontal grid row. The first horizontal row will correspond to the determined coordinate increments x, y, z and the actual thermophysical coefficients are assigned to it. In the following rows, their values are determined on the basis of the average thickness change in accordance with the coordinate z depending on the spatial coordinate increment y.

In the mathematical module, there is used the rectangular coordinate system, in which different discretization in accordance with the coordinates x, y, and z is allowed. In this case, the coordinate increment is equal to Δx=0.002 m; Δy=0.01 m; Δz=0.02 m. We assign a homogeneous initial temperature distribution T=20 °C. Their values are assigned in proportion to the actual areas from the first column of the arrays of thermophysical coefficients per each row of the units.

**Assessment of Factors Influence**

The model configuration was developed for a bus front brake unit (Ga=16000 kg). The width of the brake lining (friction belt) is equal to 0.08 m. On
this friction belt section, the thermal conductivity coefficient values are changed in proportion to the actual areas in the model and the values of the boundary conditions of the third kind are assigned in the same way on the model boundaries.

Generally, the thermal behavior of the brake units is influenced by a great number of factors that are presented in the work [5]. The expert assessment of these factors provided a possibility to make a conclusion about the fact that the friction surface temperature of the disc brake \( T=f(Q, \alpha, h) \), where \( Q \) – thermal flow density, \( \text{W/m}^2 \); \( \alpha \) – coefficient of heat transfer \( \text{W/(m}^2 \cdot \text{deg.}) \); \( h \) – disc thickness, m. Thus, it is necessary to study the influence of these factors on the disc brakes temperature when conducting the 1\(^{st}\) type tests on the basis of statistical analysis of the design parameters and probable values of the heat transfer coefficients.

In view of this condition, the experiment planning should be used in the disc brakes in order to obtain the multifactor mathematical model of the thermal process [10]. The planning matrix of the 2\(^{nd}\) type experiment is provided in the table.

<table>
<thead>
<tr>
<th>No.</th>
<th>Factors values</th>
<th>Temperature T, °C</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( X_1 (Q, \text{kw/m}^2) )</td>
<td>( X_2 (\alpha, \text{W/m}^2 \cdot \text{deg.}) )</td>
<td>( X_3 (h, \text{m}) )</td>
<td>Ventilated discs</td>
</tr>
<tr>
<td></td>
<td>experimental</td>
<td>theoretical</td>
<td>experimental</td>
<td>theoretical</td>
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<tr>
<td>1</td>
<td>1 ( \cdot 10^6 )</td>
<td>50</td>
<td>0,046</td>
<td>163</td>
</tr>
<tr>
<td>2</td>
<td>2 ( \cdot 10^6 )</td>
<td>50</td>
<td>0,046</td>
<td>306</td>
</tr>
<tr>
<td>3</td>
<td>1 ( \cdot 10^6 )</td>
<td>75</td>
<td>0,046</td>
<td>145</td>
</tr>
<tr>
<td>4</td>
<td>2 ( \cdot 10^6 )</td>
<td>75</td>
<td>0,046</td>
<td>269</td>
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<td>2 ( \cdot 10^6 )</td>
<td>75</td>
<td>0,062</td>
<td>273</td>
</tr>
</tbody>
</table>

Based on the computer experiment data processing, the regression formula for determination of the friction surfaces temperatures of the MV ventilated disc brakes was obtained at the end of the previous stage of the 1\(^{st}\) type tests:

\[
T=221,88+67,37X_1–13,62X_2+1,13X_3–4,62X_1 \cdot X_2+0,62X_1 \cdot X_3+0,12X_2 \cdot X_3+0,12X_1 \cdot X_2 \cdot X_3, \quad (9)
\]

where formulas \( X_1=\frac{Q–1,5 \cdot 10^6}{0,5 \cdot 10^6} \); \( X_2=\frac{\alpha–62,5}{12,5} \); \( X_3=\frac{h–0,054}{0,008} \).

The obtained study results allow to state that the suggested models of the 2\(^{nd}\) type experiment describe the process adequately since there can be seen a non-significant difference between the computer experiment and theoretical temperature values. This indicates a necessity to proceed to higher order model, i.e. to the square-full model of the 3\(^{rd}\) type.

**Summary and conclusions.**

It is shown that the square-full model should be used based on the conducted analysis and study of the thermal models of the vehicle disc brakes.

**References**

1. DSTU UN/ECER 13-09-2002. The only technical regulations regarding the official approval of road vehicles of categories M, N, About braking. (UNECE Rules