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# STRENGTH OF THE WEB OF I-BEAMS OF REINFORCED CONCRETE UNDER THE ACTION OF SHEAR FORCE

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***Abstract.*** *All existing methods for calculating the strength of reinforced concrete elements are based on experimentally established possible types of element failure under certain forces caused by external forces. The resistance of reinforced concrete elements to shear force differs significantly from other forces, as it is characterized by different types of failure, one of which is the failure of the webs of I-beams in the strip between inclined cracks as a result of concrete fragmentation. Existing methods for calculating strength in this type of failure are far from perfect and are mainly empirical in character.*

*This paper presents a method for calculating the strength of I-beam reinforced concrete webs when they fail along a strip between inclined cracks, based on the principles of reinforced concrete plasticity theory, considering the concrete of the web under conditions of plane stress compression-tension when tensile stresses are transferred to the concrete from shear reinforcement. Within the framework of the developed method, the criterion for failure is taken to be the attainment of the main compressive stresses in the concrete strip of the web at the corresponding stress state. The general case of the ultimate equilibrium of a beam within the strip between inclined cracks with arbitrary content and location of transverse reinforcement is considered. Calculated dependencies are obtained for calculating the limit values of stresses in the strip and shear force at wall failure.*

***Key words:*** *concrete, theory plasticity, compression, stress state, strength, I-beam, web, failure, design.*

## Introduction.

One possible type of failure to reinforced concrete elements under shear forces is the fragmentation of concrete in an inclined strip between inclined cracks. This type of failure occurs in I-beams and I-sections, as well as in thin-webs rectangular beams. In this case, the inclined cracks between which the failure occurs are regular in nature, form in the middle of the section height and, as the load increases, develop along a straight line in the direction of the tension and compression edges. In I-beams, inclined cracks develop within the webs, usually up to the boundaries of the flanges in the tensile and compressive zones.

The concrete of the beam webs between the inclined cracks is under flat stress conditions of compression and tension, which is characterized by axial compression along the axis of the strip and tension transmitted to the concrete by the transverse reinforcement located at an angle. Within the framework of existing calculation methods, the strength of concrete in the strip between inclined cracks, which determines the strength of the element itself, is usually determined very approximately on the basis of empirical dependencies that do not take into account the actual stress-strain state of the concrete strip and, first of all, its shear reinforcement.

This work is a development of the provisions of the theory of plasticity of reinforced concrete [1,2] as applied to the calculation of the strength of concrete in the strip between inclined cracks in I-beams based on a flat stress state of compression-tension with the transfer of tensile forces to the concrete from the reinforcement.

**Main text**

In [2], based on the theory of reinforced concrete plasticity [1], various cases of flat stress state of reinforced concrete are considered, including the case that occurs in the webs of I-beams under the action of shear forces, namely, the stress state of compression-tension with the transfer of tensile stresses to concrete from reinforcement located at an angle to the direction of compressive stresses (Figure 1).

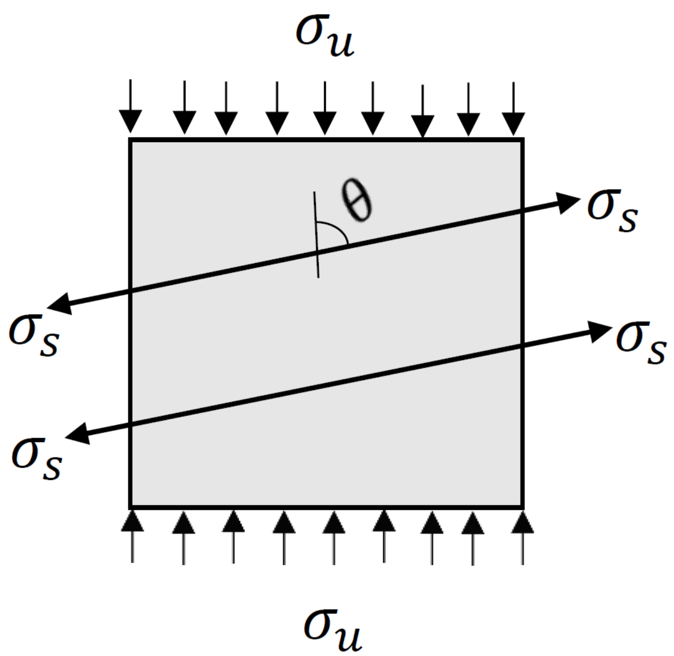


**Figure 1 - Diagram of the flat stress state of reinforced concrete under compression and tension when tensile stresses are transferred to the concrete from the reinforcement**

The value of the ultimate compressive stress $\sigma_u$ in the case under consideration is calculated using the formula:

$$\sigma_u = \rho \cdot \sigma_s(cos\,\theta - sin\,\theta)\,cos^2\,\theta + \\ + \frac{f_c - f_{ct} + \rho \cdot \sigma_s \cdot sin^2\,\theta \cdot (sin\,\theta - \sigma_s/f_y)}{2} + \\ + \sqrt{\begin{aligned}&\left(\frac{f_c + f_{ct}}{2}\right)^2 + 0{,}75 \cdot \rho \cdot f_y \cdot sin^2\,\theta \cdot (sin\,\theta - \sigma_s/f_y) \cdot \\ &\cdot \left[2(f_c - f_{ct}) - \rho \cdot f_y \cdot sin^2\,\theta \cdot (sin\,\theta - \sigma_s/f_y)\right]\end{aligned}} \quad (1)$$

The design model of an I-beam element failing as a result of concrete fragmentation between inclined cracks is shown in Figure 2.

In relation to the accepted calculation model (Figure 2), equation (1) for calculating the ultimate compressive stresses in the web of an I-beam takes the form:

$$\sigma_{cm,u} = \rho \cdot \sigma_{sw} \cdot (cos\,\theta - sin\,\theta)\,cos^2\,\theta + \\ + \frac{f_c - f_{ct} + \rho \cdot \sigma_{cm} \cdot sin^2\,\theta \cdot (sin\,\theta - \sigma_{sw}/f_y)}{2} + \\ + \sqrt{\begin{aligned}&\left(\frac{f_c + f_{ct}}{2}\right)^2 + 0{,}75 \cdot \rho \cdot f_y \cdot sin^2\,\theta \cdot (sin\,\theta - \sigma_{sw}/f_y) \cdot \\ &\cdot \left[2(f_c - f_{ct}) - \rho \cdot f_y \cdot sin^2\,\theta \cdot (sin\,\theta - \sigma_{sw}/f_y)\right]\end{aligned}} \quad (2)$$

where

$\theta$ – acute angle between the direction of action of the main compressive stresses $\theta_m$ and the transverse reinforcement $\theta_s$ (Figure 2a):

at $\theta_m \geq \theta_s$

$$\theta = \theta_m - \theta_s \quad (3)$$

at $\theta_m < \theta_s$

$$\theta = \pi/2 - \theta_m + \theta_s \quad (4)$$

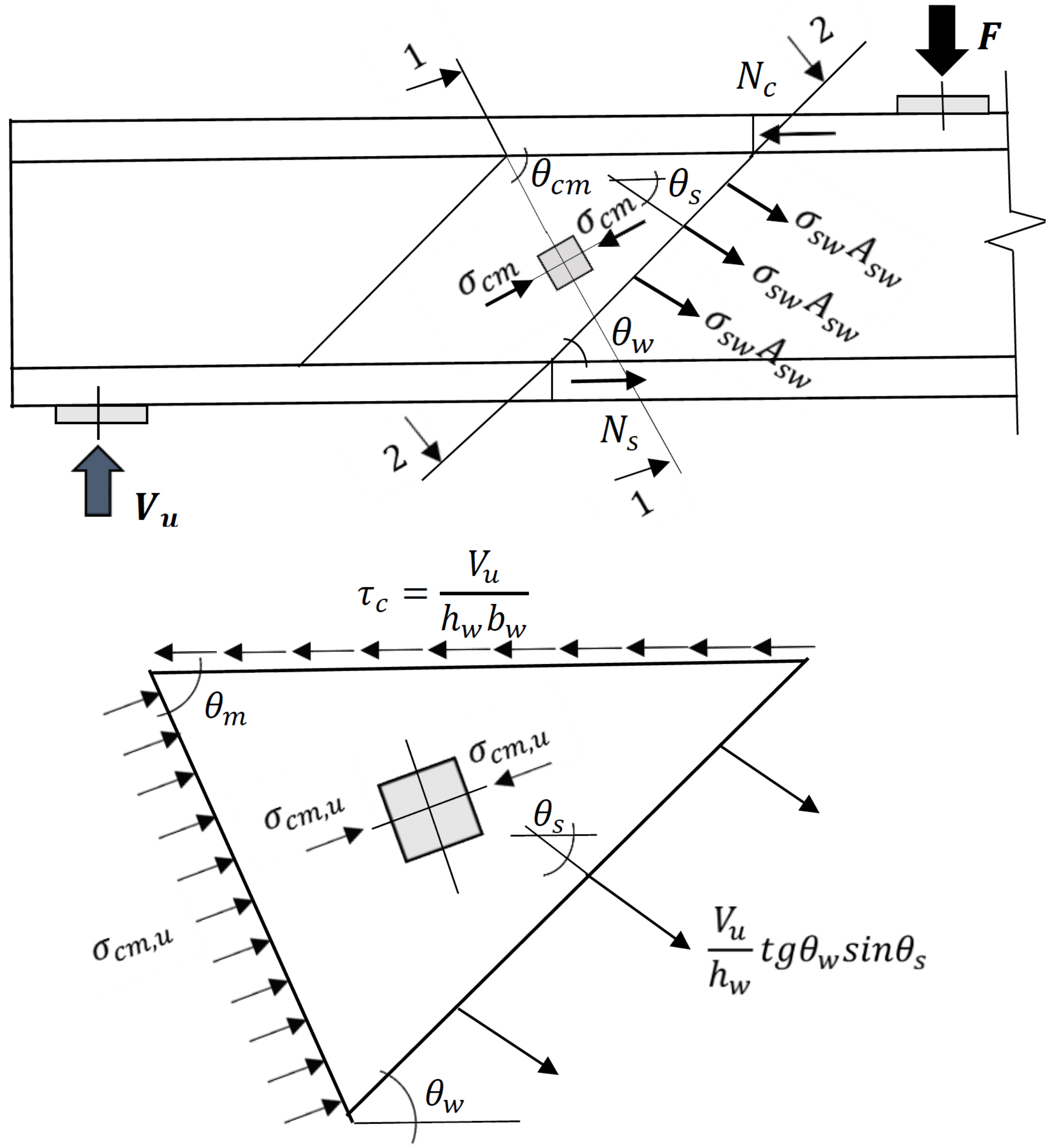


**Figure 2 - Design model of an I-beam element failure due to concrete fragmentation in the zone between inclined cracks: a -** *distribution of external and internal forces*; **b -** *distribution of forces in the strip*

The ultimate shear force experienced by the element is determined from the equilibrium of the strip, selected by section 1-1 (Figure 2b) by:

$$V_u = \sigma_{cm,u} \cdot b_w \cdot h_w \frac{sin\,\theta_w}{cos(\theta_m - \theta_w)[sin\,\theta_m - cos\,\theta_m \cdot tg(\theta_m - \theta_s)} \quad (5)$$

Angle of inclination of the main compressive stress action areas in the concrete strip $\theta_m$ is found from the equilibrium condition, assuming that the tangential stresses in section 1-1 (Figure 2b) are zero according to:

$$\frac{cos\,\theta_m}{cos(\,\theta_m-\theta_s)}=sin\,\theta_w\cdot sin\,\theta_s\cdot[cos_w+sin_w\cdot tg(\theta_m-\theta_w)\quad(6)$$

In general, the sequence of calculations for determining the ultimate shear force at failure of the web of I-beam elements as a result of concrete fragmentation in the strip between inclined cracks under a given loading scheme, element geometry, transverse reinforcement, and strength characteristics of concrete and reinforcement can be represented as follows:

- the calculation is performed by sequentially considering a series of crack inclination angles in the range $1,0 \leq ctg\theta_w \leq 2,5$;

- for each inclination angle, according to (6), the areas of main compressive stresses $\theta_m$ are taken;

- according to (2), the value of the main compressive stresses $\sigma_{cm,u}$ is calculated within the framework of a conservative approach, assuming that the stresses in the shear reinforcement are equal to the yield strength ($f_y$); otherwise, the stresses in the shear reinforcement can be determined from the element equilibrium equation in section 2-2 (Figure 2a), assuming;

- according to (5), the ultimate shear force perceived by the element at failure $V_u$ is calculated.

Figure 3 shows an example of the application of the developed calculation method for an I-beam with concrete strength $f_c$=20MPa, with transverse reinforcement $\rho$ =0,001 with strength $f_y$=400MPa in the vertical position of the ties, and graphs comparing the ultimate shear force as a function of the angle of inclination of the strip between the inclined cracks are constructed for calculations using the developed method and EN 1992-1-1:2023 [3]. The analysis of Figure 6 shows that the developed method generally correctly estimates the trends in the change in the ultimate shear force from the angle of inclination of the strip, while the calculated values exceed the corresponding values according to EN 1992-1-1:2023 [3]. The latter corresponds to the conclusions [4], according to which the experimental value of the ultimate shear force perceived by the element at an angle of inclination of the transverse reinforcement of $45^0$ can exceed the calculated value according to [3] by 2...3.5 times.

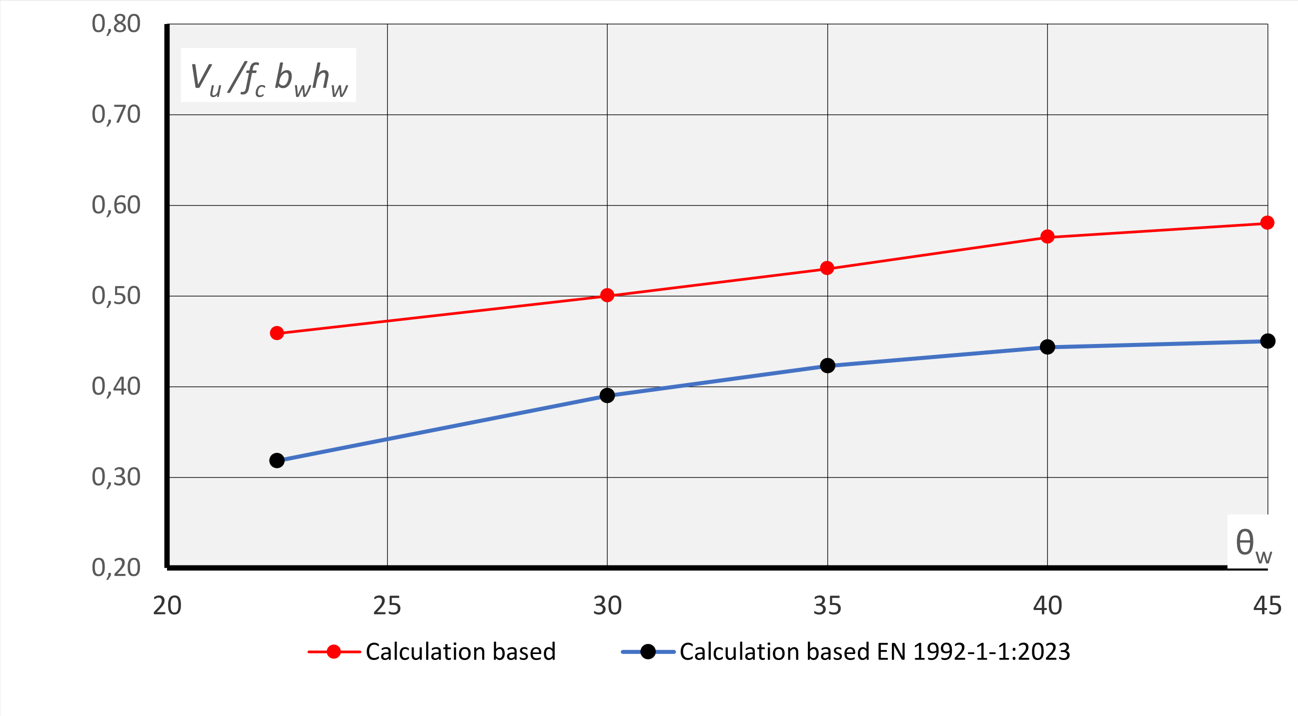


**Figure 3 - Dependence of the calculated ultimate shear force on the angle of inclination of the strip**

**Summary and conclusions**.

This paper presents methods for designating the strength of elements with transverse reinforcement under the action of longitudinal compressive forces on the webs of T-beams under the action of shear forces, developed on the basis of a unified model of reinforced concrete plasticity theory.

The method of calculating the strength of centrally compressed reinforced concrete elements with transverse mesh reinforcement presented in this article is based on, that failure of the element occurs when the ultimate state is reached in the concrete core and transverse reinforcement, which is considered to be an internal connection that causes a restriction of transverse deformations of concrete and, as a result, the occurrence of triaxial compression in concrete. Based on the theory of concrete plasticity, formulas for calculating the limit values of stresses in concrete, taking into account the content of transverse reinforcement, were obtained, and a comparison with experimental data was performed, revealing a sufficiently high accuracy of the developed method for calculating the strength of an element.

The method of calculating the strength for design strength of webs of reinforced concrete I-beams in case of failure along a strip between inclined cracks, based on the provisions of the theory of plasticity of reinforced concrete, considering the concrete of the web under conditions of plane stress compression-tension when tensile stresses are transferred to the concrete from shear reinforcement. Within the framework of the developed method, the criterion for failure is taken to be the attainment of the main compressive stresses in the concrete strip of the web to their ultimate values under the corresponding stress state. The general case of the ultimate equilibrium of a beam within the strip between inclined cracks with arbitrary content and location of shear reinforcement is considered. Calculated dependencies for determining the ultimate stress values in the strip and the shear force at web failure are obtained. A comparison was made between the calculations using the developed method and EN 1992-1-1:2023 [3], establishing the correctness of the developed method in reflecting the main trends in the change in the ultimate shear force perceived by the element, which allows the developed method to be further extended to the calculation of thin-walled T-shaped and rectangular beams.

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