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APPLICATION OF THE SMALL PARAMETER METHOD IN NONLINEAR OSCILLATIONS

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Abstract *The article is about the motion of a nonlinear material system is considered, which is described by a nonlinear differential equation that is explicitly independent of time. Application of the small parameter method was shown an example, were considered the oscillations of a nonlinear system - a mathematical and physical pendulums at large amplitudes.*

Key words: *small parameter, nonlinear material system, nonlinear oscillations.*

Introduction.

Already in the last century, there was a mathematical apparatus that, with proper development and generalization, could be applicable to the study of nonlinear oscillations, in any case, for oscillations close enough to linear ones. Sufficiently close to linear oscillations are usually called oscillations for which the corresponding differential equation, although it is nonlinear, contains some parameter ε included in this equation so that when this parameter is equal to zero, the nonlinear differential equation degenerates into a linear one with constant coefficients. It is assumed that the parameter μ is "small".

Statement and solution of the problem

Consider the motion of a non-linear material system, which is described by a non-linear differential equation that does not explicitly depend on time:

$$\ddot{x}+k^2x=\mu f(x) \tag{1}$$

where μ is some parameter, which is a coefficient for a nonlinear continuous differentiable function *f(x)*. It is assumed that the parameter μ is sufficiently small.

We write the periodic solution in the form [1]

$$x=x_0+\mu x_1+\mu^2 x_2+\ldots \tag{2}$$

here x_0, x_1, x_2 are unknown periodic functions of the circular frequency p and frequencies that are multiples of p, which are to be determined. We represent p_2 as a *p*-polynomial in powers of the small parameter μ.

$$p^2=k^2+b_1\mu_1+b_2\mu_2^{\ 2}+\ldots$$

Here are b_1,b_2 are constant coefficients determined in the process of integration equation (1), their values are chosen in such a way that solution (2) is periodic, in which there are no so-called. "resonant" terms, unlimited increasing over time. Substituting solution (2) into equation (1), we obtain a system of inhomogeneous differential equations of the second order, in which the inhomogeneity is function of previous solutions. As an example, consider the oscillations of a nonlinear system - a mathematical or physical pendulums at large amplitudes, differential equations which are of the following form:

$$\varphi + k^2 sin\varphi = 0 \tag{3}$$

Equation (3) is non-linear. Expand sinφ into a power series.

$$\sin\varphi = \varphi + \frac{\varphi^3}{3!} + \frac{\varphi^5}{5!} + ...$$

As an example, let us restrict the series to two terms, i.e.

$$\sin\varphi = \varphi - \frac{\varphi^3}{6}$$

then equation (3) takes the form

$$\varphi = k^2 + \mu\varphi^3 = 0 \tag{4}$$

where $\mu = \frac{k^2}{6}$ is the small parameter

Let us represent the solution of Eq. (4) as a series in a small parameter, with up to terms containing m to the first power, then

$$\varphi(t) = \varphi_0(t) + \mu\varphi_1(t) \tag{5}$$

$$p^2 = k^2 + \mu b_1 \tag{6}$$

Substituting (5) and (6) into equation (4) and limiting the expansion to terms, containing a small parameter μ to the first power, and equating to zero the terms of the equation that are free from μ, as well as the coefficient in bracket at μ , we obtain a system of differential equations [2].

$$\varphi + p^2\varphi_0 = 0 \tag{7}$$

$$\varphi + p^2\varphi_0 = b_1\varphi_0 + \varphi_0^3 \tag{8}$$

with initial conditions :

$$\varphi(0) = a_0, \varphi(0) = 0 \tag{9}$$

As is known, the solution of equation (7) under the appropriate initial conditions has the form

$$\varphi_0 = a_0 \cos pt \tag{10}$$

To integrate equation (8), we introduce (10) into its right side and get

$$\varphi_1 + p^2\varphi_1 = a_0(b_1 + \frac{3}{4}a_0^2)\cos pt + \frac{1}{4}a_0^3 \cos pt \tag{11}$$

To avoid the state of resonance, we equate the coefficient

$$(b_1 + \frac{3}{4}a_0^2) = 0 \tag{12}$$

Equation (11) will now take the form

$$\varphi_1 + p^2\varphi_1 = \frac{1}{4}a_0^3 \cos pt \tag{13}$$

We find the solution of the inhomogeneous equation (13) as the sum of two solutions: private and general:

$$\varphi_1 = \overset{1}{\varphi_1} + \overset{2}{\varphi_1} \tag{14}$$

In this case

$$\overset{1}{\varphi_1} = D_1 \cos pt + D_2 \sin pt$$

the constant Φ is determined by the substitution into equation (13), whence

$$A = -\frac{A_0^3}{32p^2}$$

then the general solution of Eq. (14) can be written as [2]

$$\overset{1}{\varphi}_1 = D_1 \cos pt + D_2 \sin pt - \frac{a_0^3}{32p^2} \cos 3pt \tag{15}$$

By the initial conditions (9), we determine the constants D1 and D2, then the second approximation will take the form

$$\varphi_1 = \frac{a_0^3}{32p^2} (\cos pt - \cos 3pt) \tag{16}$$

To determine the desired law of oscillations ϕ, as well as the circular frequency p we use the results (10), (12) and (16):

$$\varphi = a_0 \cos pt + \frac{1}{192} \left(\frac{k}{p} \right)^2 a_0^3 (\cos pt - \cos 3pt) \tag{17}$$

$$p^2 = k^2 (1 - \frac{1}{8} a_0^2) \tag{18}$$

$$p = k (1 - \frac{1}{8} a_0^2)^{0,5} \tag{19}$$

Expanding (19) according to Newton's binomial formula, we obtain

$$p = k (1 - \frac{1}{16} a_0^2) \tag{20}$$

Summary and conclusions.

As follows from formula (20), the pendulum oscillates according to formula (17) with circular frequency p, which depends on the initial deviation a0, i.e. the frequency depends on the initial conditions and therefore the oscillations are not isochronous.

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